EE-565 – W8
INDUCTION MACHINE – SCALAR CONTROL

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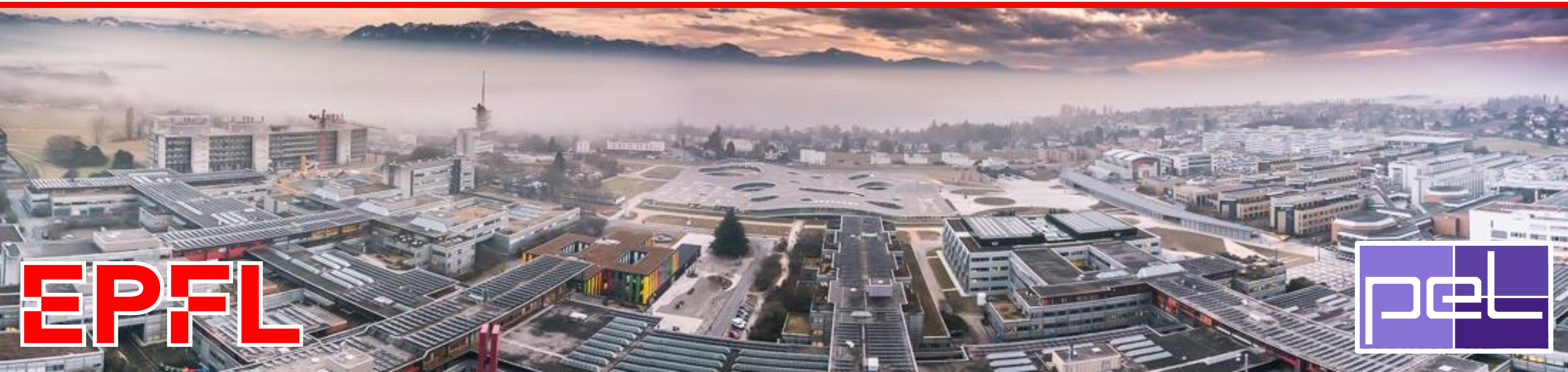


INDUCTION MACHINE
STEADY STATE OPERATION

Operation in steady state conditions

INDUCTION MACHINE MODEL – MODEL IN A COMMON REFERENCE FRAME

Electrical Equations

$$\begin{cases} \underline{v}_s = R_s \cdot \underline{i}_s + \dfrac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} + \mathrm{j}\omega_d \underline{\phi}_s \\ 0 = \underline{v}_r = R_r \cdot \underline{i}_r + \dfrac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r \\ \underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r \\ \underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s \end{cases}$$

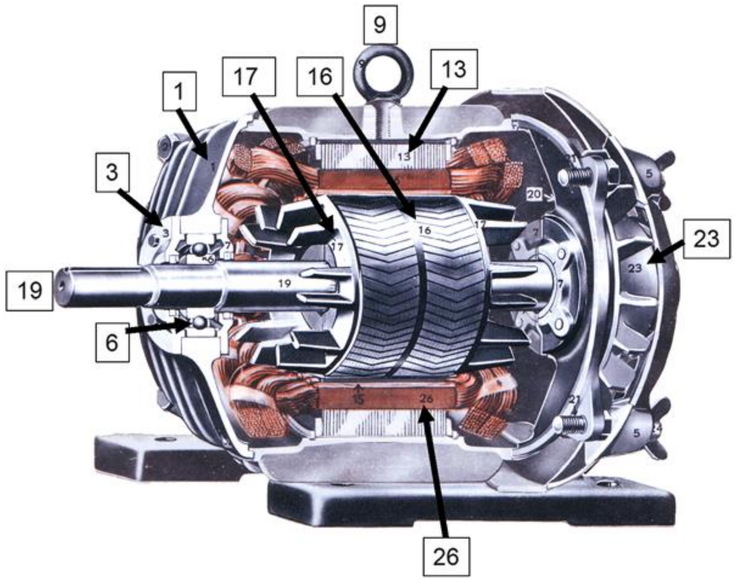
Mechanical Equations

$$\begin{cases} \dfrac{\mathrm{d}\theta_m}{\mathrm{d}t} = \omega_m \\ J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{d}t} + F(\omega_m) \cdot \omega_m = T_{em} - T_m \\ T_{em} = \dfrac{3}{2} P_p L_m \operatorname{Im}\left\{\underline{i}_s \cdot \hat{\underline{i}}_r\right\} \quad \text{(or equivalent)} \end{cases}$$

Electrical Angle Definition

$$\theta_e = P_p \theta_m$$
$$\omega_e = P_p \omega_m$$

The subscripts <dq> have been removed to simplify the notation



STEADY-STATE OPERATION

The analysis of the steady state operation is helpful to derive:

- an equivalent model for the induction machine
- the mechanical characteristics
- power balance and losses

Consider the **Stator (Stationary) reference frame** $\theta_d = 0 \qquad \omega_d = 0$

In steady state operation:

- **All the electric variables** (both in the stator and in the rotor) are **rotating vectors** at angular frequency ω_s (**synchronous speed**)
- We can identify the space vectors through the **phasors** (standard time-domain phasors)

$$\underline{v}_s = \bar{V}_s\,\mathrm{e}^{\mathrm{j}\omega_s t} \qquad \underline{i}_s = \bar{I}_s\,\mathrm{e}^{\mathrm{j}\omega_s t} \qquad \underline{\phi}_s = \bar{\Phi}_s\,\mathrm{e}^{\mathrm{j}\omega_s t} \qquad \underline{i}_r = \bar{I}_r\,\mathrm{e}^{\mathrm{j}\omega_s t} \qquad \underline{\phi}_r = \bar{\Phi}_r\,\mathrm{e}^{\mathrm{j}\omega_s t}$$

(Note: also the rotor variables are rotating at ω_s because the model is in the common reference frame of the stator)

- The derivatives of the fluxes are easily computed (the phasors are constant):

$$\frac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} = \frac{\mathrm{d}(\bar{\Phi}_s\,\mathrm{e}^{\mathrm{j}\omega_s t})}{\mathrm{d}t} = \mathrm{j}\omega_s\bar{\Phi}_s\,\mathrm{e}^{\mathrm{j}\omega_s t} \qquad \frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} = \frac{\mathrm{d}(\bar{\Phi}_r\,\mathrm{e}^{\mathrm{j}\omega_s t})}{\mathrm{d}t} = \mathrm{j}\omega_s\bar{\Phi}_r\,\mathrm{e}^{\mathrm{j}\omega_s t}$$

STEADY-STATE OPERATION

The analysis of the steady state operation is helpful to derive:

- an equivalent model for the induction machine
- the mechanical characteristics
- power balance and losses

Consider the **Stator (Stationary) reference frame** $\theta_d = 0 \qquad \omega_d = 0$

In steady state operation:

- The rotor is rotating at the mechanical speed ω_m, corresponding to the electrical speed $\omega_e = P_p\omega_m$
- **The rotor speed is generally different from the synchronous speed**
- The difference is the **slip angular frequency** $\omega_{slip} = \omega_s - \omega_e$
- The **slip** is defined as the ratio of the slip frequency and the synchronous angular frequency

$$s = \frac{\omega_{slip}}{\omega_s} = \frac{\omega_s - \omega_e}{\omega_s} = 1 - \frac{\omega_e}{\omega_s}$$

STEADY-STATE MODEL IN THE STATIONARY REFERENCE FRAME

- **Stator Voltage Balance Equation**

$$\underline{v}_s = R_s \cdot \underline{i}_s + \frac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} + \mathrm{j}\omega_d \underline{\phi}_s \quad \Rightarrow \quad \bar{V}_s = R_s \cdot \bar{I}_s + \mathrm{j}\omega_s \bar{\Phi}_s$$

- **Rotor Voltage Balance Equation**

$$0 = \underline{v}_r = R_r \cdot \underline{i}_r + \frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r \quad \Rightarrow \quad 0 = \bar{V}_r = R_r \cdot \bar{I}_r + \mathrm{j}(\omega_s - \omega_e)\bar{\Phi}_r = R_r \cdot \bar{I}_r + \mathrm{j}\, s\, \omega_s \bar{\Phi}_r$$

$$0 = \frac{R_r}{s} \cdot \bar{I}_r + \mathrm{j}\omega_s \bar{\Phi}_r$$

- **Fluxes Expressions**

$$\begin{cases} \underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r \\ \underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s \end{cases} \quad \Rightarrow \quad \begin{cases} \bar{\Phi}_s = L_s \cdot \bar{I}_s + L_m \cdot \bar{I}_r \\ \bar{\Phi}_r = L_r \cdot \bar{I}_r + L_m \cdot \bar{I}_s \end{cases}$$

- **Electromagnetic Torque Expression**

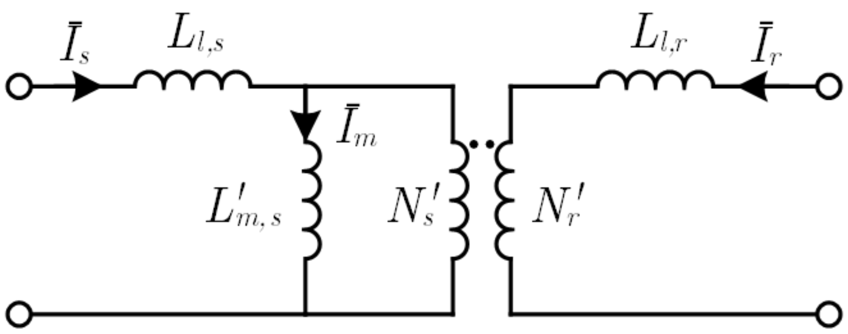
$$T_{em} = \frac{3}{2} P_p\, L_m\, \mathrm{Im}\left\{\underline{i}_s \cdot \hat{\underline{i}}_r\right\} \text{ (or equivalent)} \quad \Rightarrow \quad T_{em} = \frac{3}{2} P_p\, L_m\, \mathrm{Im}\left\{\bar{I}_s \cdot \hat{\bar{I}}_r\right\} \text{ (or equivalent)}$$

EQUIVALENT TRANSFORMER MODEL

- The expressions of the fluxes are the same of an **equivalent transformer** (with only one primary and one secondary)

$$\begin{cases} \bar{\Phi}_s = L_s \cdot \bar{I}_s + L_m \cdot \bar{I}_r \\ \bar{\Phi}_r = L_r \cdot \bar{I}_r + L_m \cdot \bar{I}_s \end{cases} \Rightarrow \begin{bmatrix} \bar{\Phi}_s \\ \bar{\Phi}_r \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} \cdot \begin{bmatrix} \bar{I}_s \\ \bar{I}_r \end{bmatrix}$$

- By using the expressions of the inductances (from the modeling lecture), we can find **the equivalent "T" model**



$$\begin{cases} L_s = L_{l,s} + 3/2\, L_{m,s} \\ L_r = L_{l,r} + 3/2\, L_{m,r} \\ L_m = 3/2\, L_{m,sr} \end{cases}$$

$$\begin{cases} L_{m,s} = \dfrac{{N'_s}^2}{R_{m,eq}} \\ L_{m,r} = \dfrac{{N'_r}^2}{R_{m,eq}} \\ L_{m,sr} = \dfrac{N'_s N'_r}{R_{m,eq}} \end{cases}$$

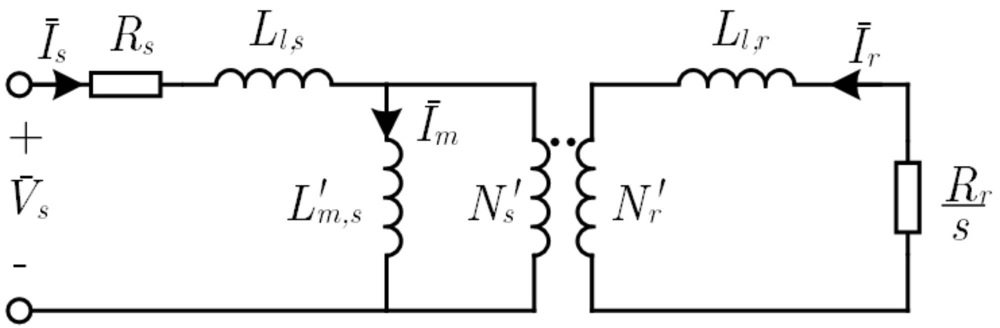
- The equivalent magnetizing inductance $L'_{m,s}$ considers the **simultaneous effect of all three phases**

$$L'_{m,s} = \frac{3}{2} L_{m,s} = \frac{3}{2} \frac{{N'_s}^2}{R_{m,eq}}$$

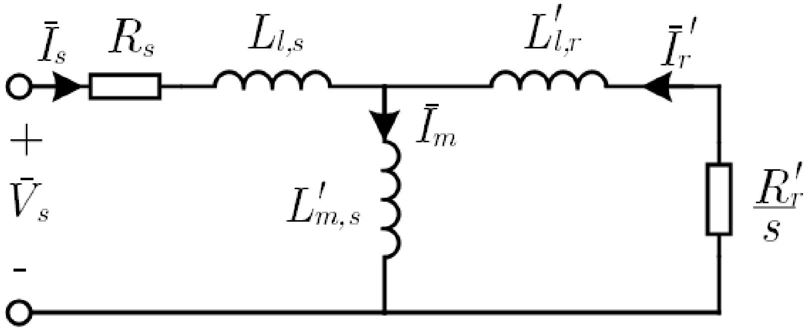
- $\bar{I}_m$ is named **magnetizing current**

EQUIVALENT CIRCUIT OF THE INDUCTION MACHINE

- By using also the voltage balance equations, an equivalent circuit of the induction machine can be found



- The rotor parameters can be referred to the stator by considering the equivalent transformation ratio N'_s/N'_r



$$L'_{l,r} = L_{l,r}\,(N'_s/N'_r)^2$$

$$R'_r = R_r\,(N'_s/N'_r)^2$$

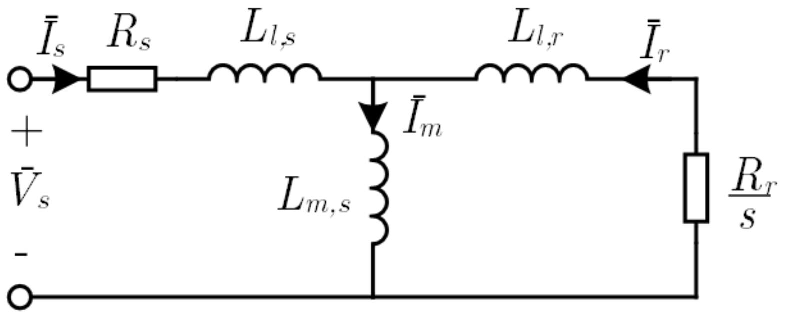
$$\bar{I}'_r = \bar{I}_r\,(N'_r/N'_s)$$

- In most cases, the rotor parameters are directly computed as referred to the stator, and N'_s and N'_r are not needed in the model
- For notation compactness, the superscript « $'$ » is often removed, and $L'_{m,s}$ is often just denoted as L_m

ANALOGY WITH TRANSFORMER

The equivalent circuit shows analogy with a transformer with:

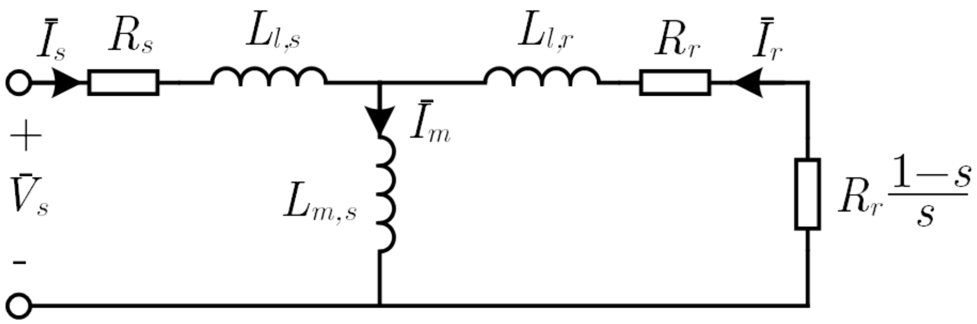
- Stator winding as primary
- Rotor winding as secondary, closed on a resistance
- Equivalent rotor resistance depending on the slip
  - with locked rotor, s=1
  - with rotor in motion, the slip is s < 1
  - at synchronysm, s = 0



The superscript « ' » has been removed to simplify the notation

The secondary resistance can be divided in two contributions:

- The rotor resistance R_r
  - Responsible for the rotor joule losses
- The equivalent mechanical resistance $R_r(1-s)/s$
  - Responsible for the power conversion (from electrical to mechanical)



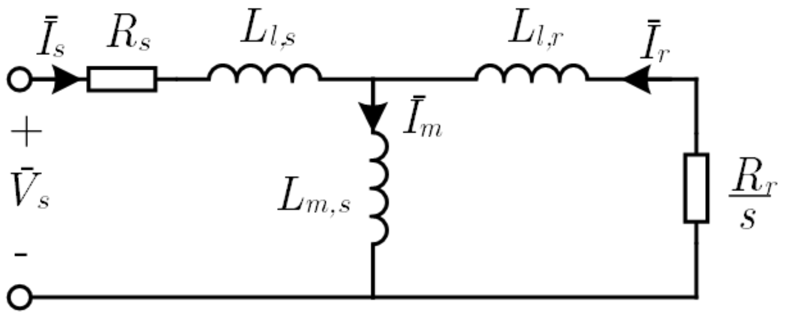
ELECTROMAGNETIC TORQUE

The active power dissipated by the equivalent mechanical resistance in the equivalent circuit is converted to mechanical power and can be used to compute the equivalent torque in steady state conditions:

$$P_{em} = \frac{3}{2} R_r \frac{1-s}{s} I_r^2$$

$$T_{em} = \frac{P_{em}}{\omega_m} = \frac{3}{2}\frac{P_p}{\omega_e} R_r \frac{1-s}{s} I_r^2$$

$$= \frac{3}{2} P_p R_r \frac{\omega_e/\omega_s}{s\,\omega_e} I_r^2 = \frac{3}{2}\frac{P_p R_r}{s\,\omega_s} I_r^2$$



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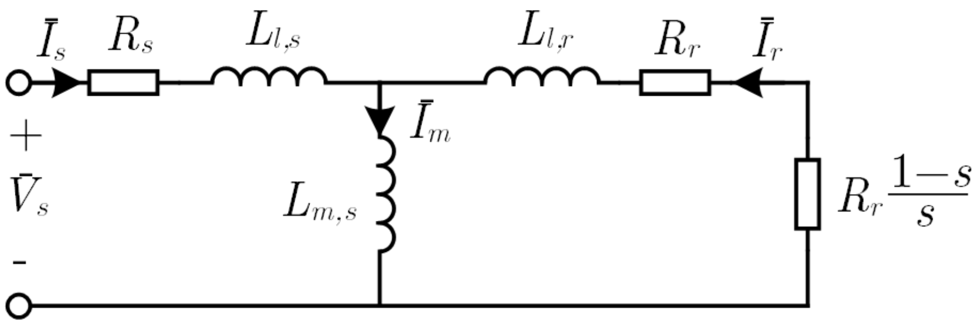
The rotor current can be found by solving the circuit for known stator voltage:

$$-\bar{I}_r = \bar{I}_s \frac{Z_m}{Z_m + Z_r} = \frac{\bar{V}_s}{Z_s + (Z_m // Z_r)} \frac{Z_m}{Z_m + Z_r}$$

$$= \frac{\bar{V}_s}{Z_s + Z_r + (Z_s\, Z_r / Z_m)}$$



with $Z_s = R_s + \mathrm{j}\omega_s L_{l,s}$ $\quad Z_r = R_r/s + \mathrm{j}\omega_s L_{l,r}$ $\quad Z_m = \mathrm{j}\omega_s L_{m,s}$

ELECTROMAGNETIC TORQUE

The electromagnetic torque can be expressed as a function of the stator current:

$$T_{em} = \frac{3}{2}\frac{P_p\,R_r}{s\,\omega_s}\,I_r^2 = \frac{3}{2}\frac{P_p\,R_r}{s\,\omega_s}\frac{(\omega_s L_{m,s})^2}{(R_r/s)^2+\omega_s^2(L_{m,s}+L_{l,r})^2}I_s^2$$

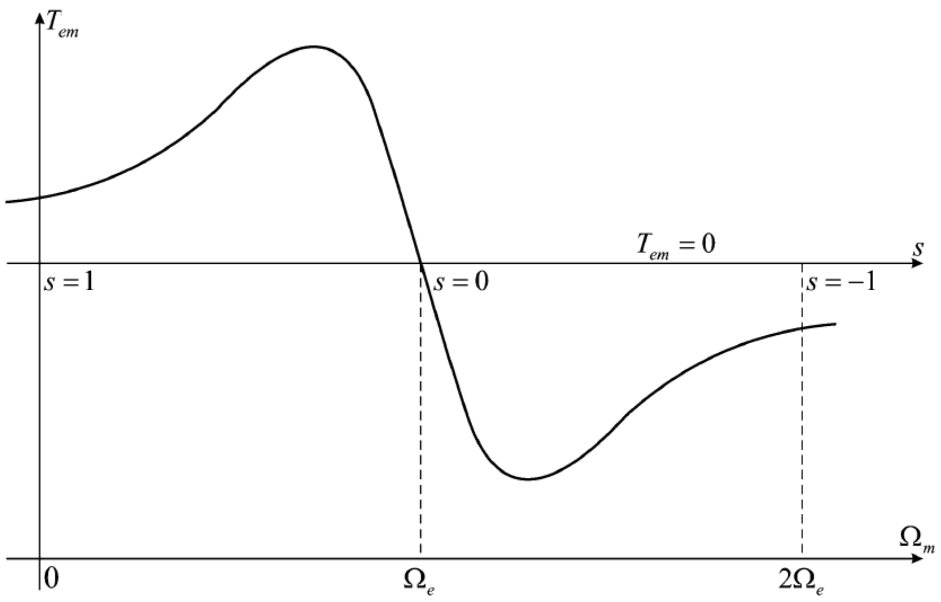
Under the assumption that the magnetizing current is negligible, the stator current squared magnitude can be approximately expressed as the rotor squared current, and obtained as:

$$I_r^2 \approx I_s^2 \approx \frac{V_s^2}{(R_s+R_r/s)^2+\omega_s^2(L_{l,s}+L_{l,r})^2}$$

Then, the electromagnetic torque can be approximately expressed as:

$$T_{em} = \frac{3}{2}\frac{P_p\,R_r}{s\,\omega_s}\,I_r^2 \approx \frac{3}{2}\frac{P_p\,R_r}{s\,\omega_s}\frac{V_s^2}{(R_s+R_r/s)^2+\omega_s^2(L_{l,s}+L_{l,r})^2}$$

This expression depends on the voltage magnitude and angular frequency



ELECTROMAGNETIC TORQUE

The developed torque depends on the slip

- For small slip values ($s \approx 0$)

$$I_r^2 \approx I_s^2 \approx \frac{V_s^2}{(R_r/s)^2}$$

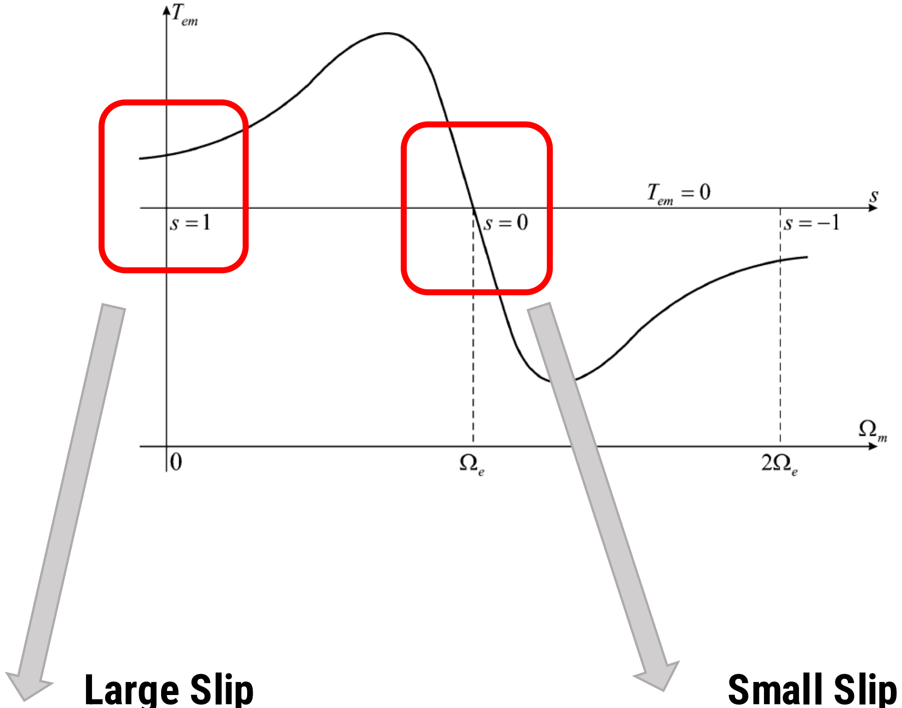
$$T_{em} \approx \frac{3}{2} P_p \frac{V_s^2}{R_r} \frac{s}{\omega_s} \approx \frac{3}{2} \frac{P_p}{R_r} \frac{V_s^2}{\omega_s^2} \omega_{slip} \approx \frac{3}{2} \frac{P_p}{R_r} \Phi_s^2 \omega_{slip}$$

- For large slip values ($s \approx 1$)

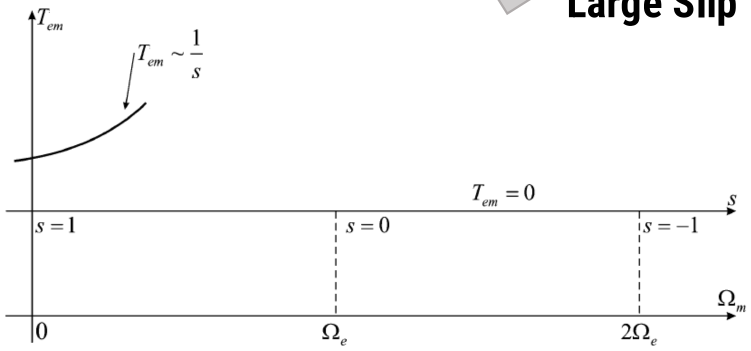
$$I_r^2 \approx I_s^2 \approx \frac{V_s^2}{\omega_s^2 L_{l,tot}^2}$$

with $L_{l,tot} = L_{l,s} + L_{l,r}$

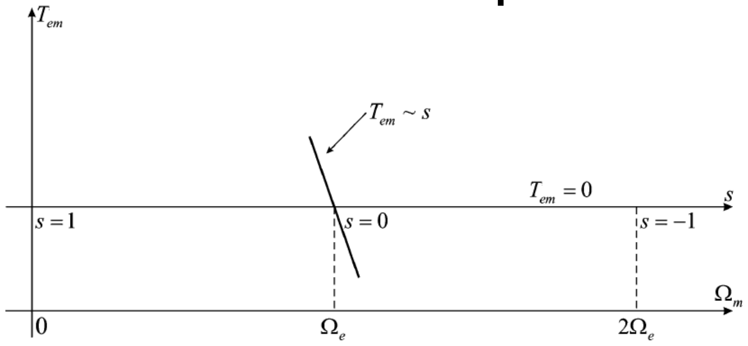
$$T_{em} \approx \frac{3}{2} P_p \frac{V_s^2}{\omega_s^3} \frac{R_r}{L_{l,tot}^2} \frac{1}{s}$$



Large Slip



Small Slip



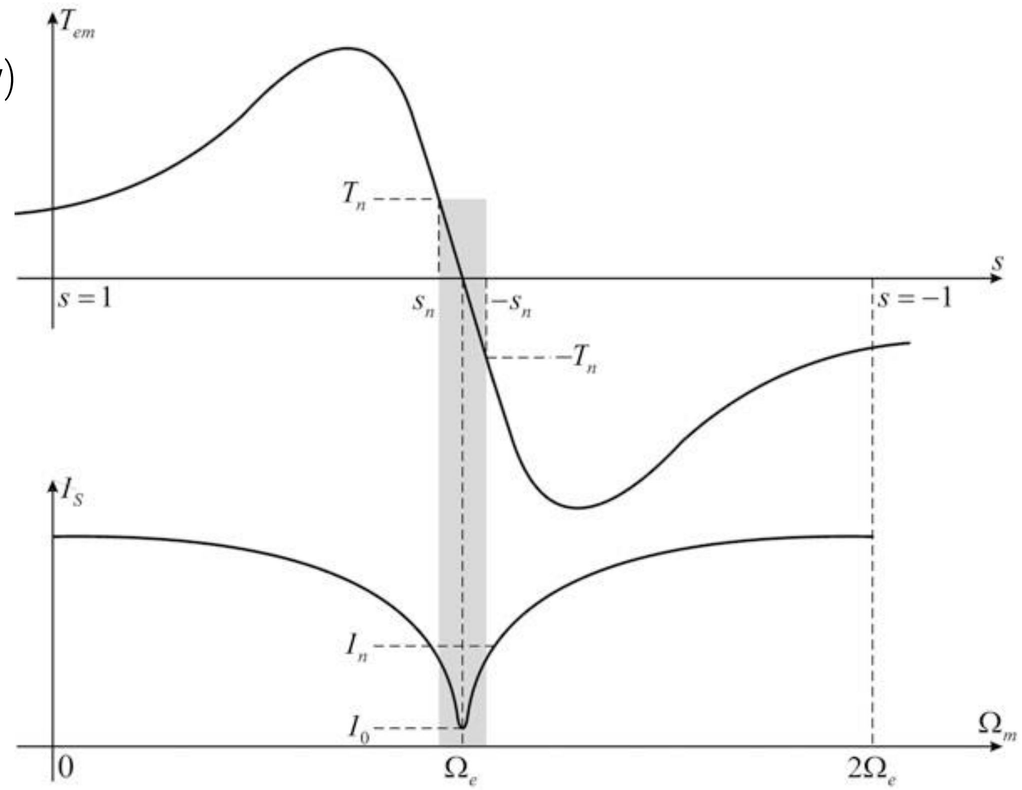
ELECTROMAGNETIC TORQUE

From the equivalent model it is also possible to compute the stator current for different rotor speed (considering fixed angular frequency of the stator voltage supply)

- **The current is decreasing for smaller values of the slip**
- At perfect synchronism ($s = 0$), only the magnetizing current is present
- At the start-up ($s = 1$), the stator current is normally very high (even 4÷8 times the nominal current, depending on the machine parameters and ratings)

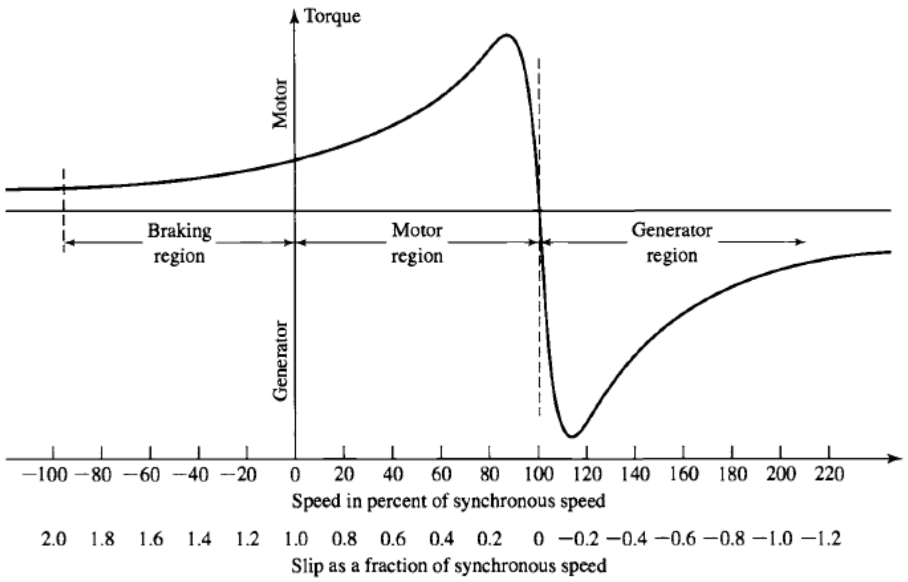
- The **maximum stator current** that can be sustained by the machine (in steady state conditions) determines **the maximum steady-state torque** that can be accepted

- Typically, **the maximum steady-state torque is limited by the maximum allowed current**, and it is smaller than the maximum electromechanical torque of the characteristics (breakdown torque)

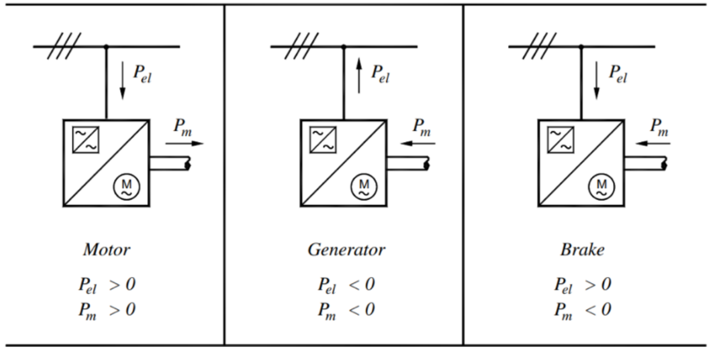


According to the slip, different operating regions can be recognized for the machine:

► **Motor Region** $(0 < s < 1)$
The angular speed of the rotor (ω_e) is slower than the angular speed of the field in the air-gap (ω_s)
The machine generates torque in the same direction of the speed (**positive mechanical power**)
The machine absorbs electrical power from the stator terminals (**positive electrical power**)
Therefore, **the machine converts electrical power into mechanical power**



► **Generator Region** $(s < 0)$
The angular speed of the rotor (ω_e) is faster than the angular speed of the field in the air-gap (ω_s)
The machine generates torque in the opposite direction than the speed (**negative mechanical power**)
The machine provides electrical power to the stator terminals (**negative electrical power**)
Therefore, **the machine converts mechanical power into electrical power**



► **Braking Region** $(s > 1)$
The rotor and the air-gap field rotate in opposite directions
The machine generates torque in the opposite direction than the speed (**negative mechanical power**)
The machine absorbs electrical power from the stator terminals (**positive electrical power**)
Therefore, **the machine absorbs power from both the electrical terminals and the mechanical ports**, and **all the power is dissipated internally** (operation to avoid)

V/F SCALAR CONTROL

Operating principle of the V/f control

SCALAR CONTROL

The ultimate goal is to achieve control over

- Flux of the machine (stator, rotor, or magnetizing)
- Electromagnetic Torque

The **Scalar Control** has been the first approach developed to control an induction machine for variable speed drives

It is based on the **control of the stator flux** through **manipulation of the stator voltage**

- **Magnitude**
- **Angular Frequency**

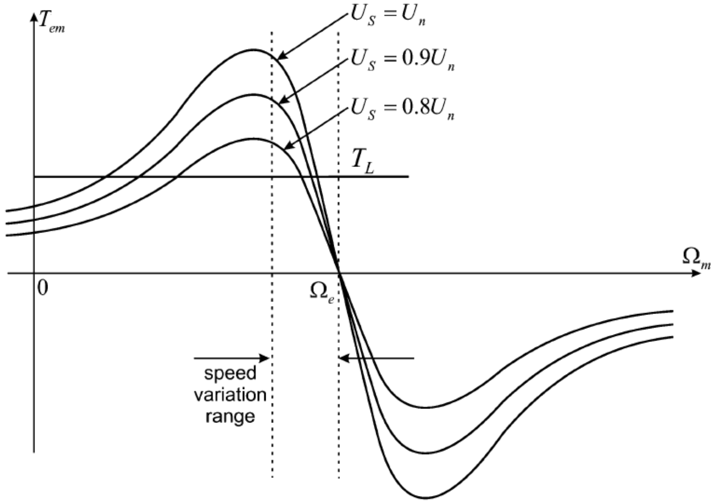
This approach is relatively simple, but does not achieve high performances

Control of flux and torque are not decoupled

V/F CONTROL LAW

For small values of the slip, the electromagnetic torque can be approximated as:

$$T_{em} \approx \frac{3}{2} P_p \frac{V_s^2}{R_r} \frac{s}{\omega_s} \approx \frac{3}{2} \frac{P_p}{R_r} \frac{V_s^2}{\omega_s^2} \omega_{slip} \approx \frac{3}{2} \frac{P_p}{R_r} \Phi_s^2 \omega_{slip}$$



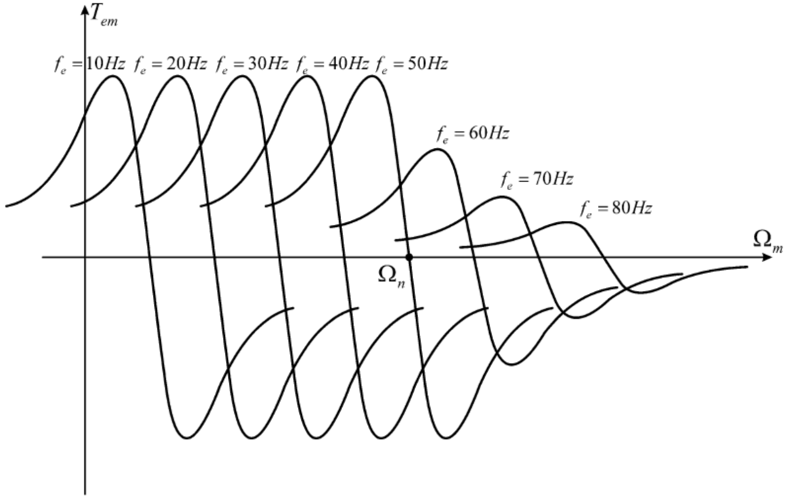
We can change the torque by either changing the flux or by changing the slip frequency

By **changing the magnitude of the flux** (at fixed frequency) we can **change the slope of the mechanical characteristic**

- ► The magnitude of the flux can be changed with the **stator voltage magnitude**

By **changing the slip frequency** (at fixed flux), we can **shift the mechanical characteristic**

- ► The slip frequency can be changed with the **angular frequency of the stator voltage supply**



V/F CONTROL LAW

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$$T_{em} \approx \frac{3}{2} P_p \frac{V_s^2}{R_r} \frac{s}{\omega_s} \approx \frac{3}{2} \frac{P_p}{R_r} \frac{V_s^2}{\omega_s^2} \omega_{slip} \approx \frac{3}{2} \frac{P_p}{R_r} \Phi_s^2 \omega_{slip}$$

We can change the torque by either changing the flux or by changing the slip frequency

The aim is to **keep the flux close to the nominal value**

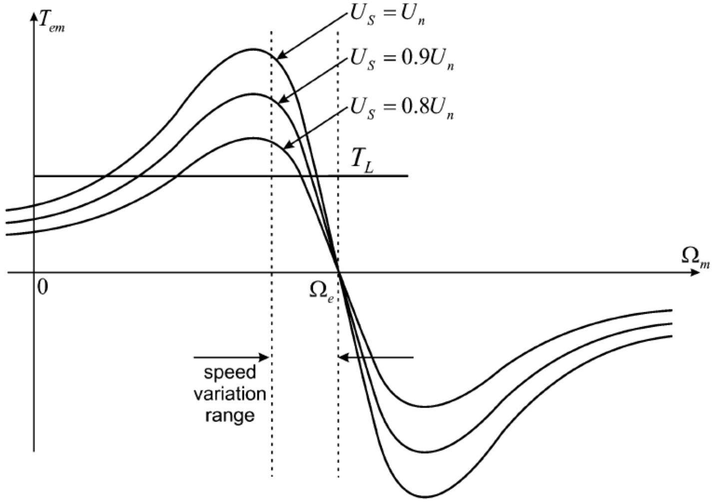
- Lower values would require higher currents to develop the same torque (higher losses)
- Higher values would lead to saturation of the magnetic circuit

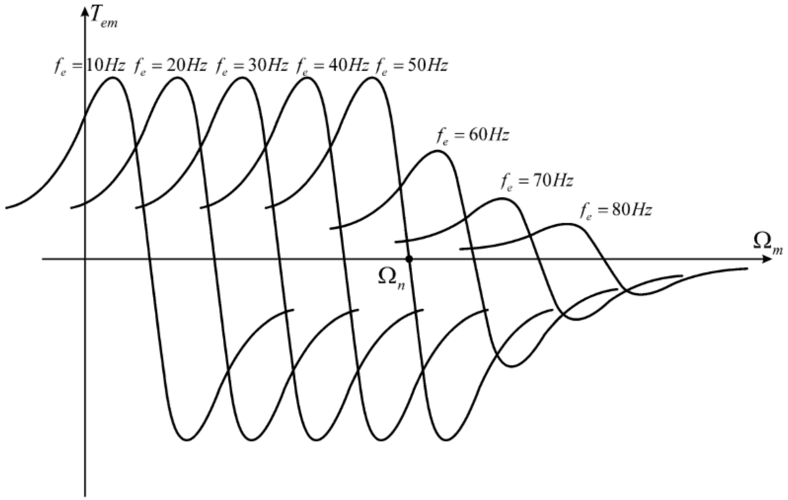
We can keep the flux constant by considering

$$\Phi_s \approx \frac{V_s}{\omega_s}$$

The **voltage magnitude** and the **angular frequency** are changed **proportionally**

This approach is commonly recognized as **V/f control law**





V/F CONTROL LAW

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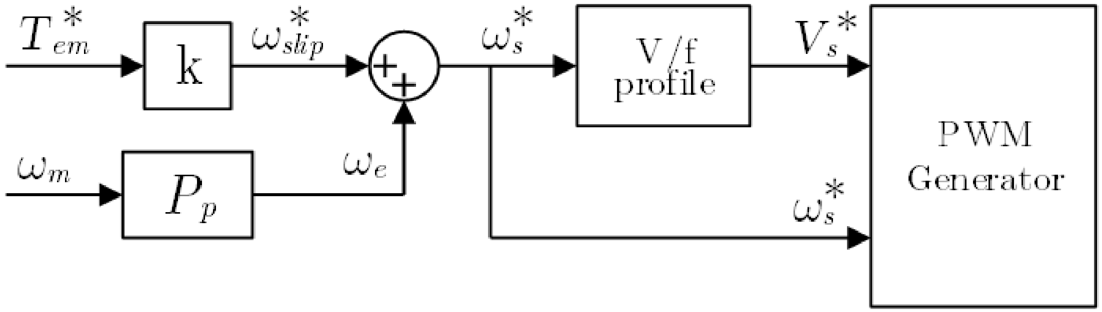
$$T_{em} \approx \frac{3}{2} P_p \frac{V_s^2}{R_r} \frac{s}{\omega_s} \approx \frac{3}{2} \frac{P_p}{R_r} \frac{V_s^2}{\omega_s^2} \omega_{slip} \approx \frac{3}{2} \frac{P_p}{R_r} \Phi_s^2 \omega_{slip}$$

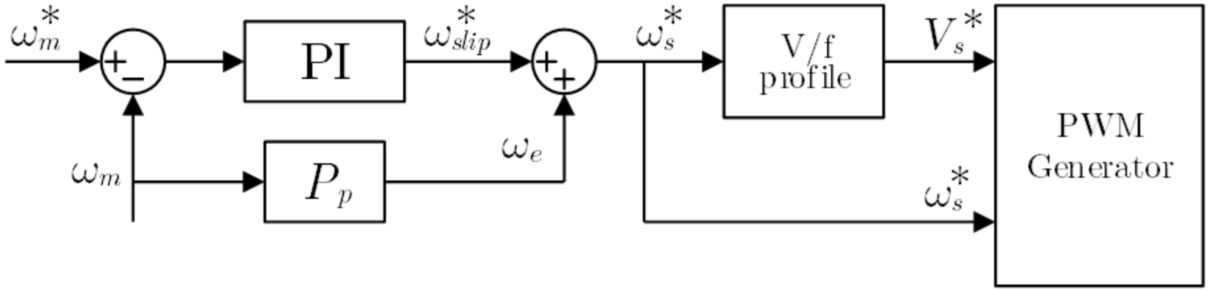


The **slip frequency** can be computed from the desired torque as:

$$\omega_{slip}^* = \frac{2}{3} \frac{R_r}{P_p} \frac{T_{em}^*}{\Phi_{s,n}^2}$$

(e.g., using the nominal flux)

Typically, the slip frequency is directly obtained from a closed-loop controller (e.g. speed controller)



The **angular frequency for the stator** can be computed as:

$$\omega_s^* = \omega_e + \omega_{slip}^* = P_p \omega_r + \omega_{slip}^*$$

The **stator voltage magnitude** can be computed from the **V/f profile** as:

$$V_s^* = \Phi_{s,n} \cdot \omega_s^* = \frac{V_{s,n}}{\omega_{s,n}} \cdot \omega_s^*$$

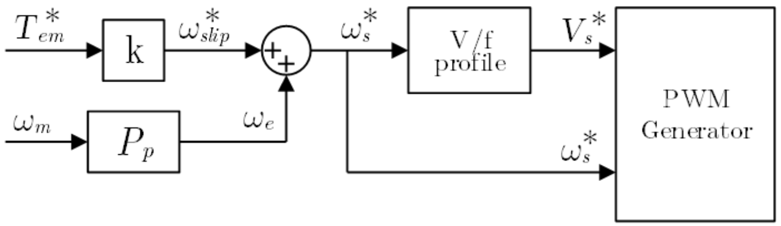
(e.g., using the nominal voltage and angular frequency)

Other V/f profiles can be used for low speeds and field weakening – discussed later

FIELD WEAKENING

The aim is to **keep the flux close to the nominal value**

- Lower values would require higher currents to develop the same torque (higher losses)
- Higher values would lead to saturation of the magnetic circuit



However, **operation at constant flux is not always possible**

The stator voltage that can be applied is limited by

- **Maximum voltage that can be supplied by the converter** (depending on the DC-bus voltage)
- **Maximum voltage for the electric insulation of the machine** (depending on the design)

Above the nominal speed, the voltage is kept constant

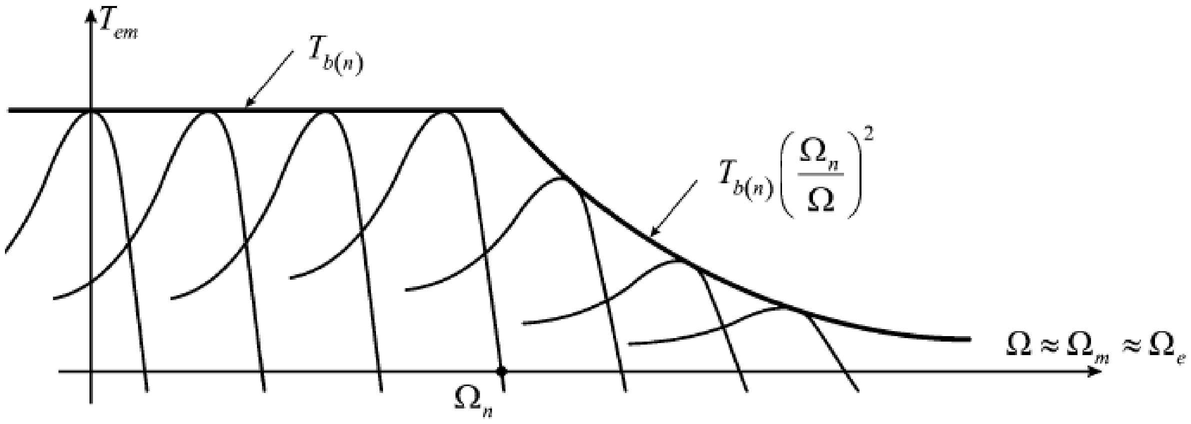
Therefore, the flux is inversely proportional to the angular speed

This operation leads to the **Field Weakening** of the machine

With the decrease of the flux, also **the maximum torque is decreased**

The machine cannot work anymore at constant torque

The machine will work at (almost) **constant power**



RESISTIVE DROP COMPENSATION

The approximation $V_s \approx \omega_s \Phi_s$ is **only valid at high frequency**

At low speed (low frequency), the resistive drop cannot be neglected compared to the electromotive force

$$\bar{V}_s = R_s \cdot \bar{I}_s + \mathrm{j}\omega_s \bar{\Phi}_s$$

If the supply voltage is varied proportionally to the angular speed, at low frequency the flux will drop below the rated valu e

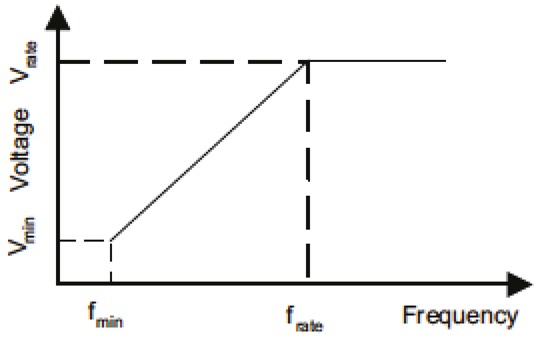
The decrease of the flux leads to a decrease in the torque that can be applied by the machine

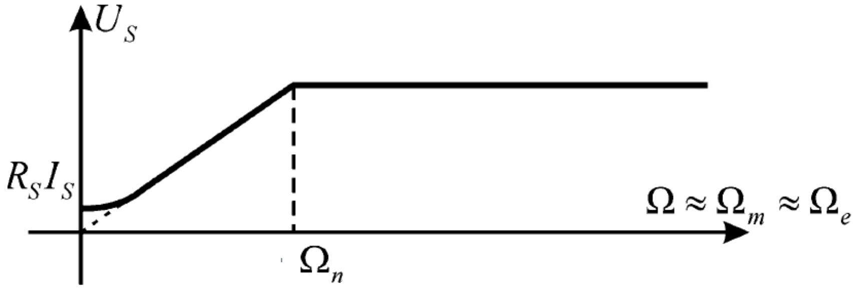
This phenomenon can be counteracted by **modifying the V/f control low at low frequency**, to take into account the resistive drop

For frequencies approaching zero, the magnitude of the flux is kept to nominal value if the voltage approaches $V_{s,0} = R_s \cdot \underbrace{I_{s0,rated}}_{\text{No load current under nominal supply}}$

Different profiles have been proposed

- ► Constant voltage below a minimum frequency
- ► Linear voltage with a different slope
- ► Parabolic voltage characteristics
- ► etc…





OPERATING LIMITS

The operating limits of the machine depend on:

- **Maximum Flux** – limited by design to avoid saturation
- **Maximum Stator Current** – limited to avoid overheating
  - A maximum steady-state current and a maximum transient current could be defined
- **Maximum Stator Voltage** – limited by DC-bus and by insulating performances

From these constraints, the machine is limited in terms of torque and power

The operating area is similar to a DC machine:

- **Below rated speed**, the flux limit and current limit are more relevant

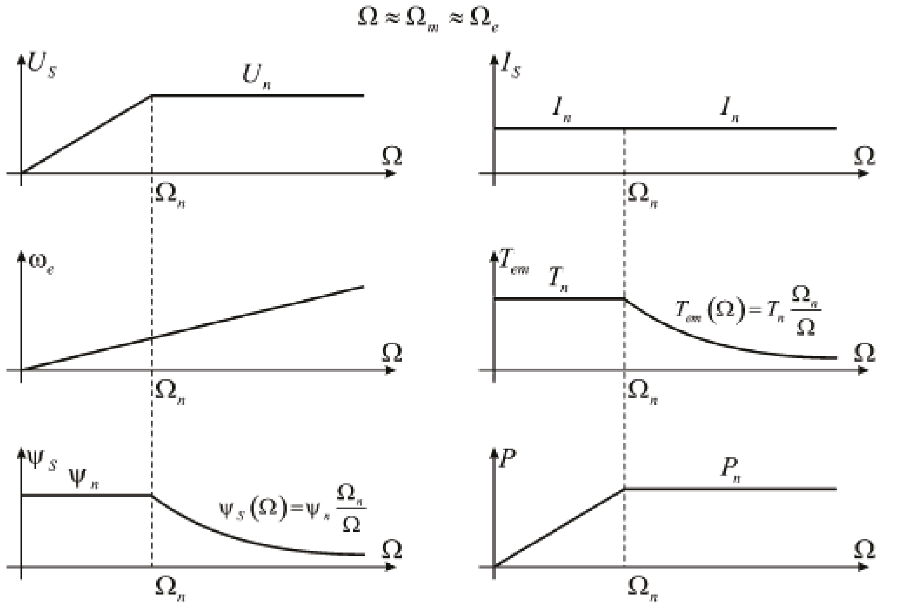
  The characteristic is (almost) at **constant maximum torque**

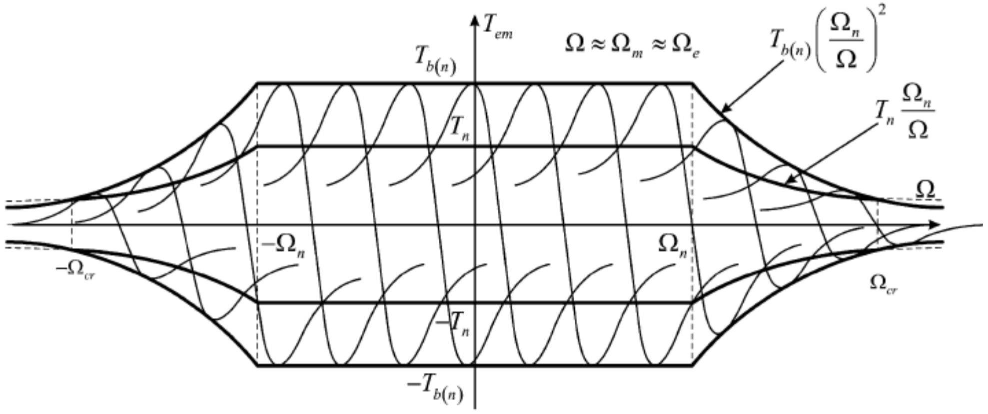
  The maximum power is proportional to the rotor speed

- **Above rated speed**, the voltage limit and current limit are more relevant

  The characteristic is (almost) at **constant maximum power**

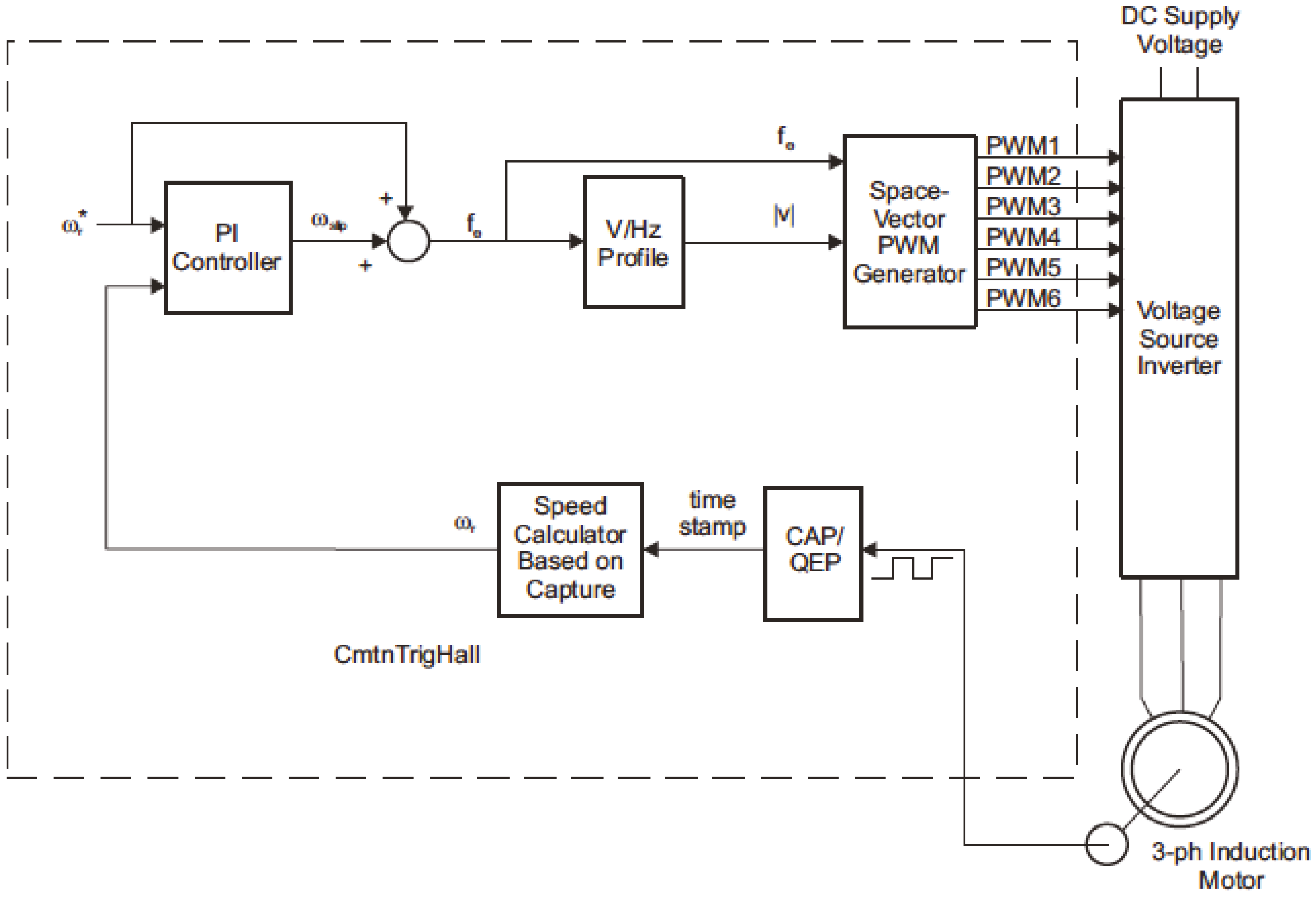
  The maximum torque is inversely proportional to the rotor speed





BLOCK DIAGRAM

- A closed-loop speed controller can be added to the control algorithm
- A standard PWM algorithm can be used to generate the switching pulses (e.g., SPWM, THIPWM, SVM, etc…)
- Since the scalar control is based on steady state characteristics of the machine, it does not provide high dynamic performances
- For tuning purposes, the time delay of the system (from voltage reference to electromagnetic torque) needs to consider the complete extinction of all the electric transients



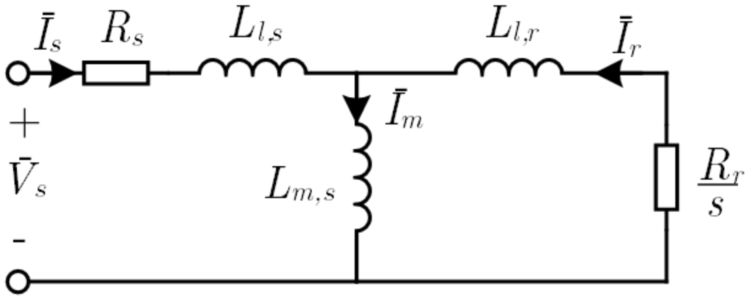
IS-BASED SCALAR CONTROL

Operating principle of the scalar control based on stator currents

SCALAR CONTROL BASED ON STATOR CURRENTS

- Instead of controlling the stator voltages**, it is possible to formulate the algorithm to control the stator currents**
- From the equivalent circuit, it is possible to easily compute the rotor currents phasor as

$$-\bar{I}_r = \frac{Z_m}{Z_m + Z_r} \cdot \bar{I}_s = \frac{\mathrm{j} s\omega_s L_{m,s}}{R_r + \mathrm{j} s\omega_s (L_{l,r} + L_{m,s})} \cdot \bar{I}_s = \underbrace{\frac{\mathrm{j} s\omega_s L_m}{R_r + \mathrm{j} s\omega_s L_r}}_{\text{With all parameters implicitly referred to the stator}} \cdot \bar{I}_s$$



- The stator flux can be expressed as

$$\bar{\Phi}_s = L_s \bar{I}_s + L_m \bar{I}_r = \left(L_s - \frac{\mathrm{j} s\omega_s L_m^2}{R_r + \mathrm{j} s\omega_s L_r} \right) \cdot \bar{I}_s$$

- The electromagnetic torque is expressed as

$$T_{em} = \frac{3}{2} \frac{P_p R_r}{s\,\omega_s} I_r^2 = \frac{3}{2} \frac{P_p R_r}{s\,\omega_s} \frac{(\omega_s L_m)^2}{(R_r/s)^2 + (\omega_s L_r)^2} \cdot I_s^2$$

- **It is possible to control flux and torque through magnitude and frequency of the stator currents**
- **A more precise torque control can be achieved**

USEFUL PARAMETERS

Some additional parameters are introduced to simplify the expressions:

- **Coupling Factor**

$$k = \frac{L_m}{\sqrt{L_s\, L_r}}$$

- **Total Leakage Factor**

$$\sigma = 1 - k^2 = 1 - \frac{L_m^2}{L_s\, L_r}$$

- **Total Leakage Inductance**

$$L_\sigma = \sigma\, L_s = \frac{L_s\, L_r - L_m^2}{L_r}$$

With small leakage inductances $L_\sigma \approx L_{l,s} + L_{l,r}$

- **Stator and Rotor Time Constants**

$$T_s = \frac{L_s}{R_s} \qquad T_r = \frac{L_r}{R_r}$$

- **Stator and Rotor Transient Time Constants**

$$T_s' = \sigma\, T_s = \sigma\, \frac{L_s}{R_s} \qquad T_r' = \sigma\, T_r = \sigma\, \frac{L_r}{R_r}$$

CONTROL OF THE STATOR FLUX THROUGH THE STATOR CURRENT MAGNITUDE

The Stator Flux can be reformulated as:

$$\bar{\Phi}_s = L_s\,\bar{I}_s + L_m\,\bar{I}_r = \left(L_s - \frac{\mathrm{j}s\omega_s L_m^2}{R_r + \mathrm{j}s\omega_s L_r}\right)\cdot \bar{I}_s$$

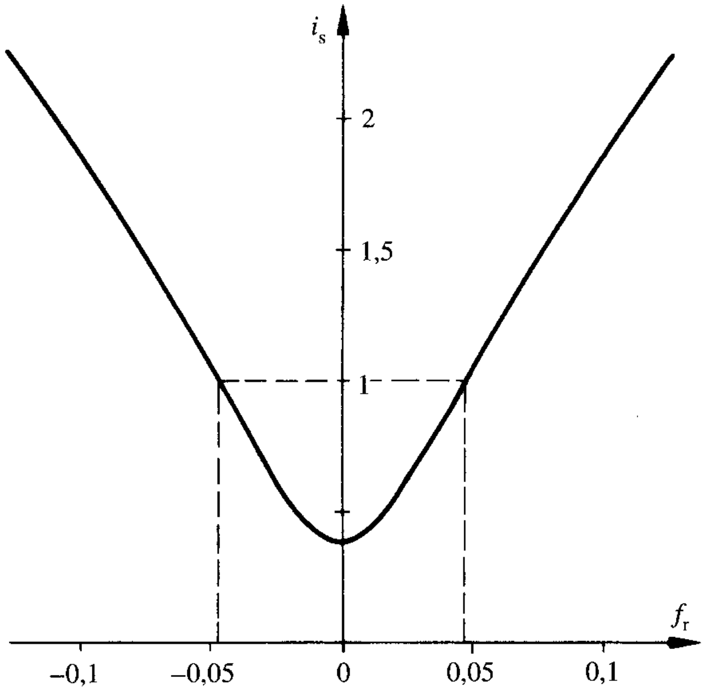
$$= L_s\left(1 - \frac{\mathrm{j}s\omega_s L_m^2/L_s}{R_r + \mathrm{j}s\omega_s L_r}\right)\cdot \bar{I}_s \quad = L_s\left(\frac{R_r + \mathrm{j}s\omega_s(L_r - L_m^2/L_s)}{R_r + \mathrm{j}s\omega_s L_r}\right)\cdot \bar{I}_s$$

$$= L_s\left(\frac{R_r + \mathrm{j}s\omega_s L_r\sigma}{R_r + \mathrm{j}s\omega_s L_r}\right)\cdot \bar{I}_s \; = L_s\left(\frac{1 + \mathrm{j}\omega_{slip}T_r'}{1 + \mathrm{j}\omega_{slip}T_r}\right)\cdot \bar{I}_s$$

The stator flux can be kept constant if the current magnitude is controlled as

$$I_s = \frac{\Phi_s}{L_s}\sqrt{\frac{1 + (\omega_{slip}T_r)^2}{1 + (\omega_{slip}T_r')^2}}$$

- It is only function of the slip frequency (i.e., rotor frequency)
- It is independent from the stator frequency



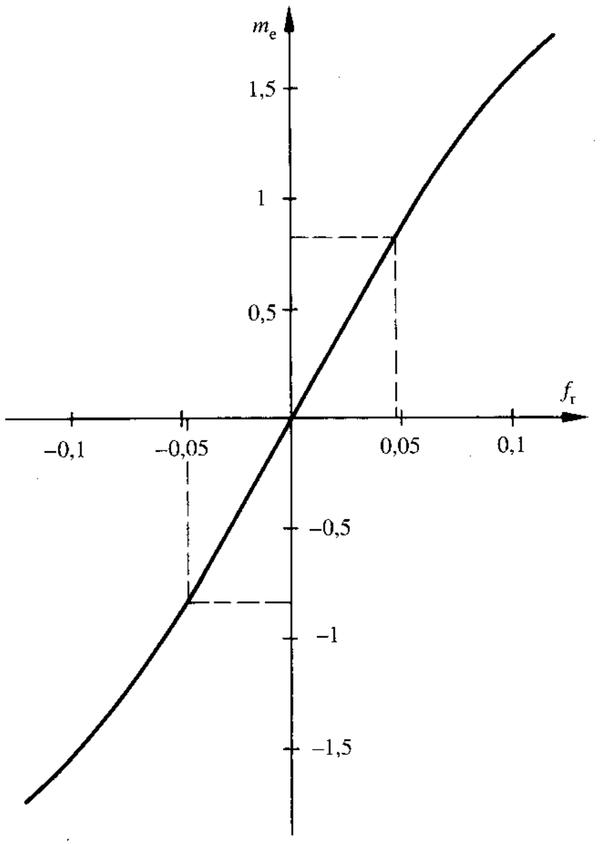
CONTROL OF THE STATOR FLUX THROUGH THE STATOR CURRENT MAGNITUDE

The electromagnetic torque can also be reformulated as:

$$T_{em} = \frac{3}{2} P_p \operatorname{Im}\left\{ \bar{I}_s \cdot \hat{\bar{\Phi}}_s \right\} = \frac{3}{2} P_p \frac{\Phi_s^2}{L_s} \operatorname{Im}\left\{ \frac{1 + \mathrm{j}\omega_{slip} T_r}{1 + \mathrm{j}\omega_{slip} T'_r} \right\}$$

$$= \frac{3}{2} P_p \frac{\Phi_s^2}{L_s} \frac{\omega_{slip}(T_r - T'_r)}{1 + (\omega_{slip} T'_r)^2} = \frac{3}{2} P_p \frac{\Phi_s^2}{L_s} \frac{\omega_{slip} T_r (1 - \sigma)}{1 + (\omega_{slip} T'_r)^2}$$

$$= \frac{3}{2} P_p \frac{\Phi_s^2}{L_s} \frac{\omega_{slip} T_r k^2}{1 + (\omega_{slip} T'_r)^2} = \frac{3}{2} P_p \frac{\Phi_s^2}{L_r} \frac{L_m^2}{L_s^2} \frac{\omega_{slip} T_r}{1 + (\omega_{slip} T'_r)^2}$$



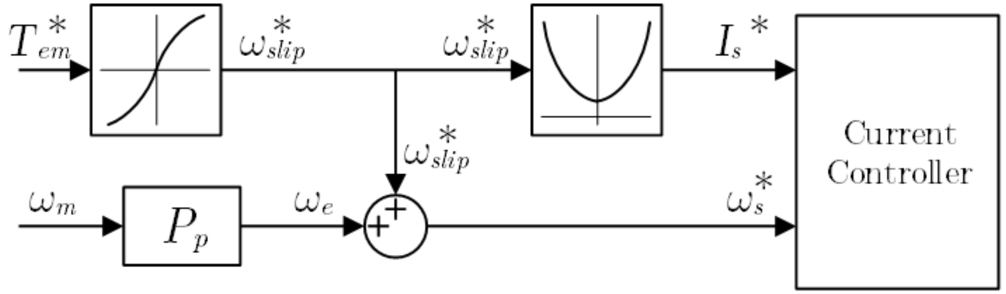
For a fixed flux, **the electromagnetic torque is only a function of the slip frequency**

- The torque changes its sign with the slip frequency (i.e., rotor frequency)
- The slip frequency is related to the applied mechanical load

The required slip frequency needed to develop a specified torque can be found by inversion of the previous formula (or by using look-up tables)

The **slip frequency** can be computed from the desired torque from inversion of:

$$T_{em} = \frac{3}{2} P_p \operatorname{Im}\left\{ \bar{I}_s \cdot \hat{\bar{\Phi}}_s \right\} = \frac{3}{2} P_p \frac{\Phi_s^2}{L_r} \frac{L_m^2}{L_s^2} \frac{\omega_{slip} T_r}{1 + (\omega_{slip} T_r')^2}$$

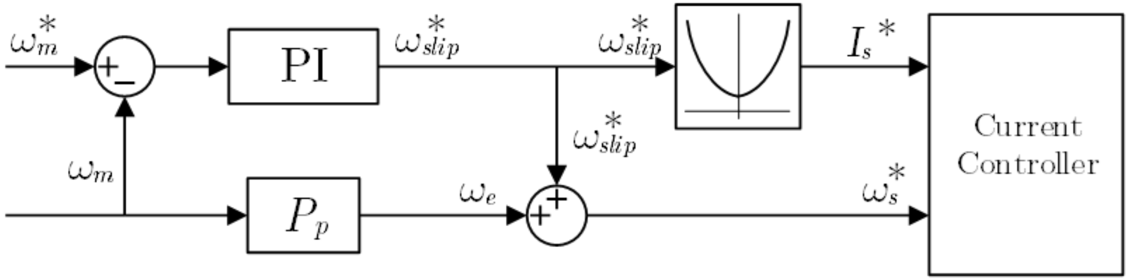


The **angular frequency for the stator** can be computed as:

$$\omega_s^* = \omega_e + \omega_{slip}^* = P_p \omega_r + \omega_{slip}^*$$

The **stator current magnitude** can be computed as:

$$I_s^* = \frac{\Phi_{s,n}}{L_s} \sqrt{\frac{1 + (\omega_{slip}^* T_r)^2}{1 + (\omega_{slip}^* T_r')^2}}$$

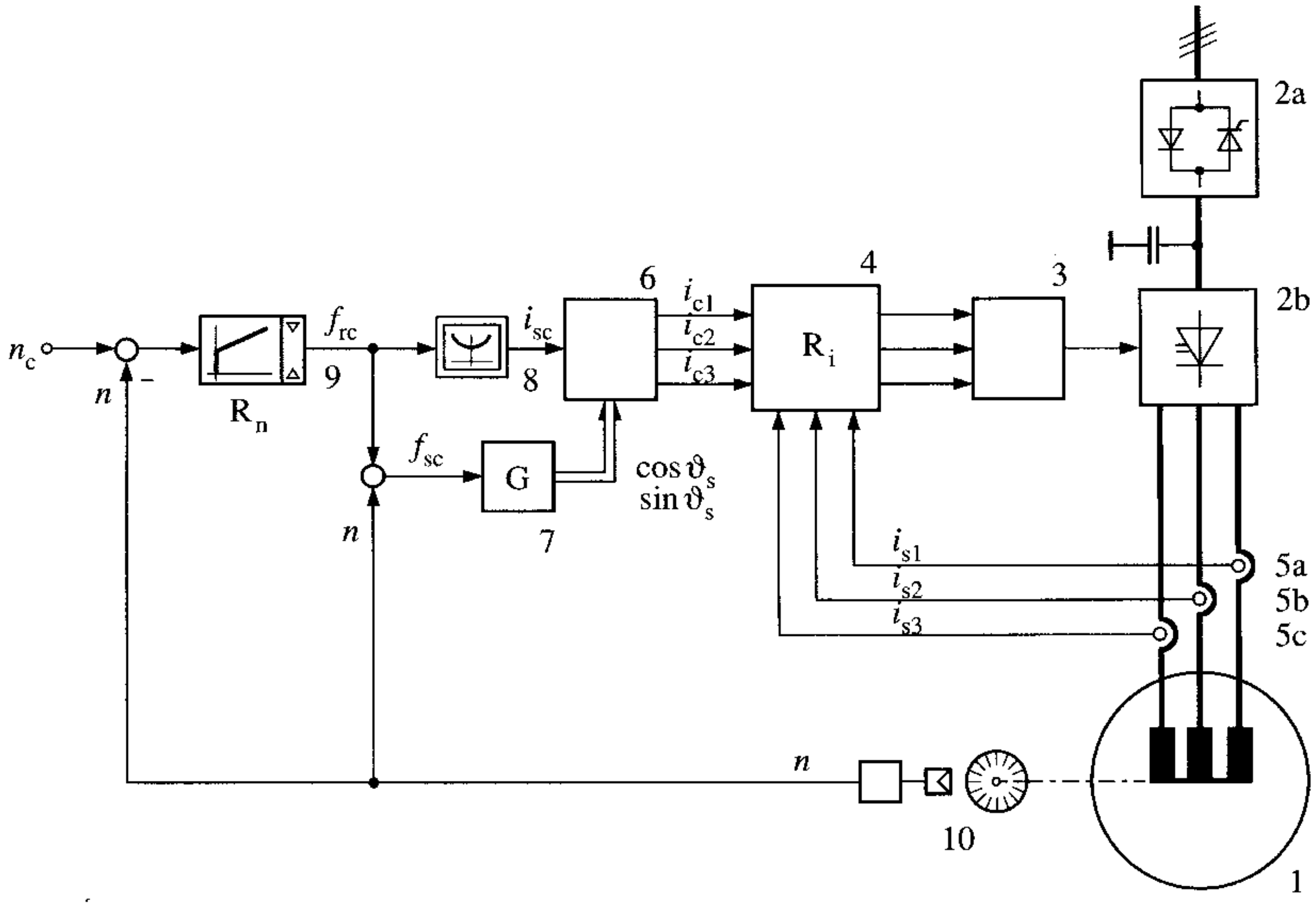


The resulting current reference must be tracked by a current controller:

- Current-Source Inverter based supply
- Closed-loop current control using a Voltage-Source Inverter
  - In the past, hysteresis controllers were used to rely on simple control without using the voltage balance equation

CONTROL OF THE STATOR FLUX THROUGH THE STATOR CURRENT MAGNITUDE

- A closed-loop speed controller can be added to the control algorithm
- Compared to V/f control, the current can be intrinsically limited by the control algorithm
- This approach does not require modifications at low speed (the flux is controlled through the currents)
- At high speed, the flux need to be decreased for the same reasons as before (field-weakening region)
- Similar control strategies have been developed to control e.g., the rotor flux to constant value

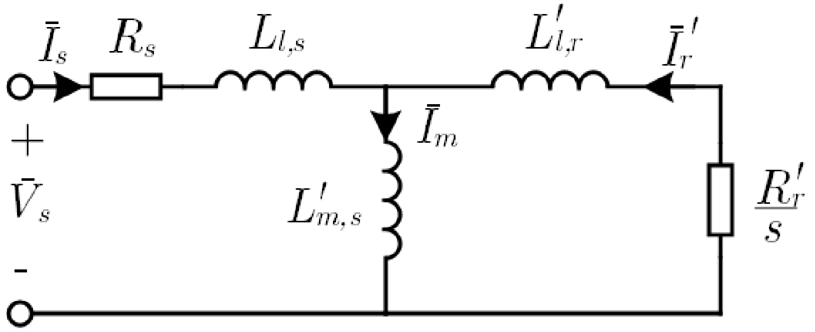


SUMMARY

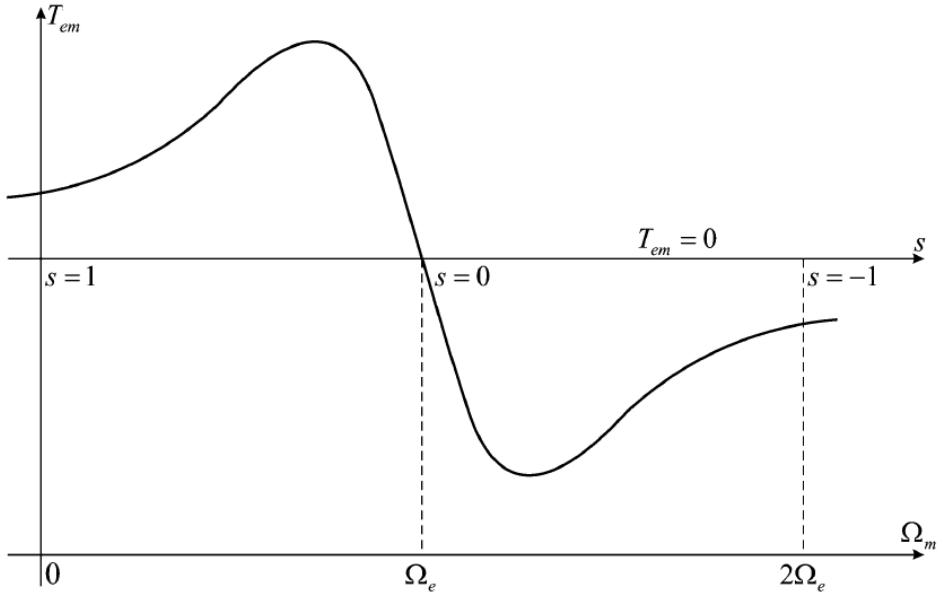
Scalar Control of the Induction Machine

STEADY-STATE OPERATION OF THE INDUCTION MACHINE

- In **steady-state** operation, all the electrical variables can be expressed by **phasors**
- A simple **equivalent circuit** can be found to analyze the system
- The equivalent circuit is **similar to the equivalent circuit of a transformer**
- The equivalent load applied at the secondary (rotor) **depends on the slip**
- Simplified expressions can be found for the electromagnetic torque

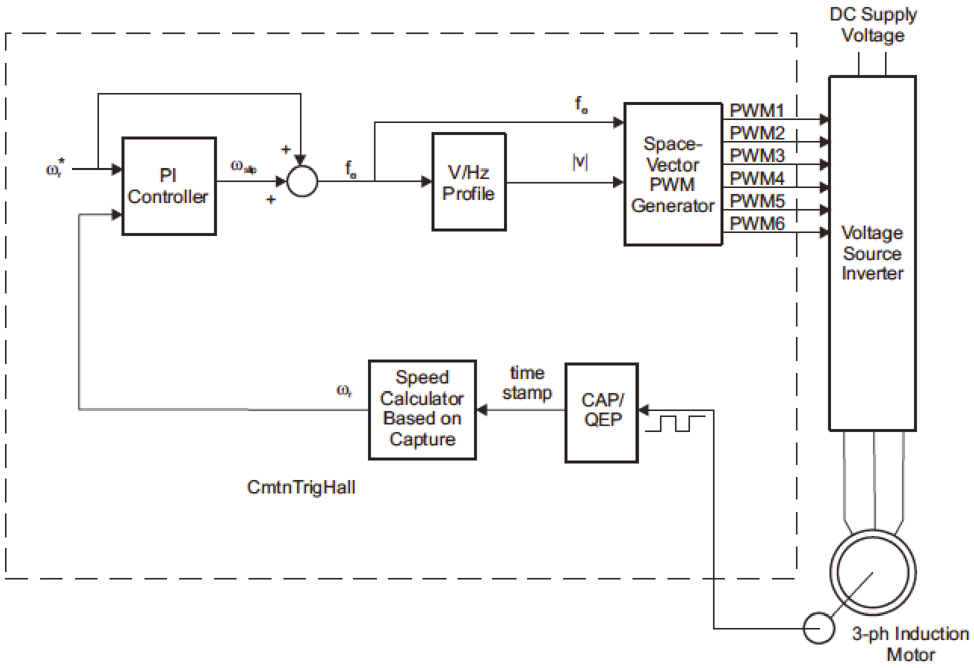


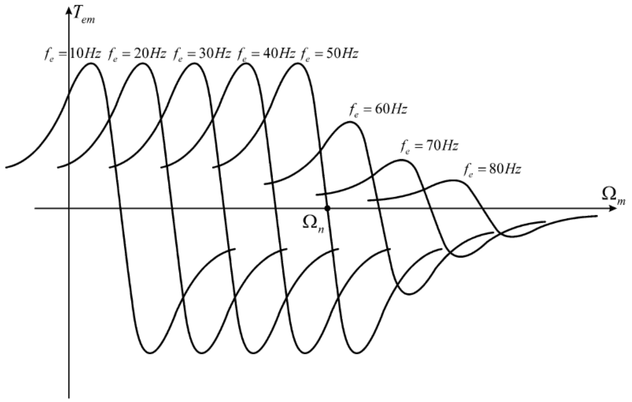
$$
\begin{cases}
\bar{V}_s = R_s \cdot \bar{I}_s + j\omega_s \bar{\Phi}_s \\
0 = \dfrac{R_r}{s} \cdot \bar{I}_r + j\omega_s \bar{\Phi}_r \\
\bar{\Phi}_s = L_s \cdot \bar{I}_s + L_m \cdot \bar{I}_r \\
\bar{\Phi}_r = L_r \cdot \bar{I}_r + L_m \cdot \bar{I}_s \\
T_{em} = \dfrac{3}{2} P_p L_m \operatorname{Im}\left\{ \bar{I}_s \cdot \hat{\bar{I}}_r \right\} \approx \dfrac{3}{2} \dfrac{P_p}{R_r} \Phi_s^2 \omega_{slip}
\end{cases}
$$

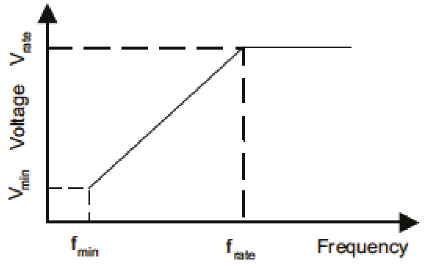


V/F SCALAR CONTROL

- The control is based on the steady-state electrical operation
- The control is based on two variables:
  - **Slip Frequency** – to control the **torque**
  - **Voltage Magnitude** – to control the **flux**
- The flux can be kept constant and equal to the rated value by **changing V and f proportionally to one another**
- For **operation at low speed**, a compensation of the resistive drops is needed, and **the V/f profile is modified**
- For **operation at high speed**, the flux need to be decreased (**field weakening**)
- **Flux and torque controls are not decoupled**, and it is not possible to achieve high dynamic performances







IS-BASED SCALAR CONTROL

- The control is based on the steady-state electrical operation
- The control is based on two variables:
  - **Slip Frequency** – to control the **torque**
  - **Current Magnitude** – to control the **flux**
- The flux can be kept constant and equal to the rated value by **controlling the stator current magnitude depending on the slip frequency**
- More precise torque control can be achieved by control of the currents
- A current-controlled supply is required
- For **operation at high speed**, the flux need to be decreased (**field weakening**)
- **Flux and torque controls are not decoupled**, and it is not possible to achieve high dynamic performances